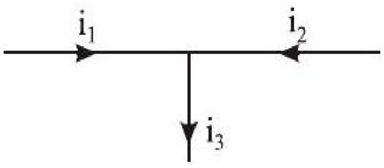
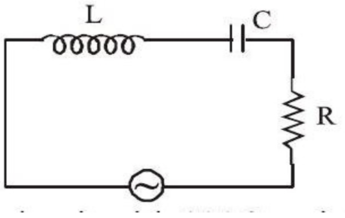
Alternating Current

1. A series AC circuit containing an inductor (20mH), a capacitor (120μ F) and a resistor (60Ω) is driven by an AC source of 24 V/50 Hz. The energy (in joule) dissipated in the circuit in 60 s is:
2. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary of the transformer is 5 A and its efficiency is 90%, the output current (in ampere) would be:
3. An alternating voltage $v(t) = 220\sin\ 100\pi t$ volt is applied to a purely resistive load of 50Ω. The time taken (in ms) for the current to rise from half of the peak value to the peak value is :
4. An inductor of inductance 100 mH is connected in series with a resistance, a variable capacitance and an AC source of frequency 2.0 kHz. What should be the value of the capacitance (in farad) so that maximum current maybe drawn into the circuit?
5. A 60 Hz AC voltage of 160 V impressed across an LR-circuit results in a current of 2 A. If the power dissipation is 200 W, calculate the maximum value of the back emf(in volt) arising in the inductance.
6. A 100 VAC source of frequency 500 Hz is connected to LCR circuit with $L = 8.1\text{mH}, C = 12.5\mu$ F and $R = 10\Omega$, all connected in series. Find the potential (in volt) across the resistance.
7. A coil has a resistance of 10Ω and an inductance of 0.4 henry. It is connected to an AC source of 6.5 V, $\frac{30}{\pi}$ Hz. Find the average power (in watt) consumed in the circuit.
8. A circuit has a resistance of 12 ohm and an impedance of 15 ohm. The power factor of the circuit will be
9. An AC generator of 220 V having internal resistance $r = 10\Omega$ and external resistance $R = 100\Omega$. What is the power (in watt) developed in the external circuit?
10. If $i_1 = 3\sin\ \omega t, i_2 = 4\cos\ \omega t$, and $i_3 = i_0\sin\ (\omega t + 53°)$, find the value of i_0.



11. Given LCR circuit has $L = 5\text{H}, C = 80\mu\ \text{F}, R = 40\Omega$ and variable frequency source of 200 V. The source frequency (in Hz) which drives the circuit at resonance is $\frac{x}{\pi}$. Find the value of x.



12. An LCR series circuit with 100Ω resistance is connected to an AC source of 200 V and angular frequency 300rad/s. When only the capacitance is removed, the current lags behind the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the current (in ampere) in the LCR circuit.

13. A current of 4 A flows in a coil when connected to a 12 VDC source. If the same coil is connected to a 12 V, 50rad/sAC source, a current of 2.4 A flows in the circuit. Determine the inductance (in henry) of the coil.

14. A capacitor of capacitance $12.0 \mu F$ is joined to an AC source of frequency 200 Hz. The rms current in the circuit is 2.00 A. Find the rms voltage (in volt) across the capacitor.

15. ALCR circuit has $L = 10\mathrm{mH}, R = 3\Omega$, and $C = 1, \mu$ F connected in series to a source of $15\cos\ \omega t$ volt. Calculate the current amplitude (in ampere) at a frequency that is 10% lower than the resonant frequency.

SOLUTIONS

1. **(5.17×10^2)** Given: $R = 60\Omega$, $f = 50$ Hz, $\omega = 2\pi f = 100\pi$ and

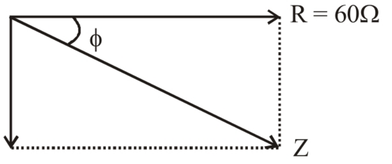
$v = 24v$

$C = 120\ \mu f = 120 \times 10^{-6} f$

$$x_C = \frac{1}{\omega C} = \frac{1}{100\pi \times 120 \times 10^{-6}} = 26.52\Omega$$

$x_L = \omega L = 100\pi \times 20 \times 10^{-3} = 2\pi\Omega$

$x_C - x_L = 20.24 \approx 20$



$$z = \sqrt{R^2 + (x_C - x_L)^2}$$

$$z = 20\sqrt{10}\Omega$$

$$\cos\phi = \frac{R}{z} = \frac{60}{20\sqrt{10}} = \frac{3}{\sqrt{10}}$$

$$P_{avg} = VI\cos\phi, I = \frac{v}{z} \quad = \frac{v^2}{z}\cos\phi = 8.64 \text{ watt}$$

Energy dissipated (Q) in time $t = 60$s is

$Q = P.t = 8.64 \times 60 = 5.17 \times 10^2$J

2. **(45)** Efficiency, $\eta = \frac{P_{out}}{P_{in}} = \frac{V_s I_s}{V_p I_p}$

$$\Rightarrow 0.9 = \frac{230 \times I_s}{2300 \times 5}$$

$$\Rightarrow I_s = 0.9 \times 50 = 45\,A$$

Output current = 45A

3. **(3.3)** As $V(t) = 220 \sin 100\pi t$

so, $I(t) = \frac{220}{50} \sin 100\pi t$

i.e., $I = I_m = \sin(100\pi t)$

For $I = I_m$

$$t_1 = \frac{\pi}{2} \times \frac{1}{100\pi} = \frac{1}{200} \text{ sec.}$$

and for $I = \frac{I_m}{2}$

$$\Rightarrow \quad \frac{I_m}{2} = I_m \sin(100\,\pi t_2)$$

$$\Rightarrow \quad \frac{\pi}{6} = 100\,\pi t_2 \quad \Rightarrow \quad t_2 = \frac{1}{600} s$$

$$\therefore \quad t_{req} = \frac{1}{200} - \frac{1}{600} = \frac{2}{600} = \frac{1}{300} s = 3.3 \text{ ms}$$

4. **(63×10^{-9})** For the maximum current,

$$X_C = X_L$$

or $$\frac{1}{\omega C} = \omega L$$

$$\therefore \quad C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L}$$

$$= \frac{1}{(2\pi \times 2 \times 10^3)^2 \times 100 \times 10^{-3}}$$

$$= 63 \times 10^{-9} F$$

5. **(125)** The impedance, $Z = \frac{V_{rms}}{i} = \frac{160}{2} = 80\,\Omega$

We know that, Power,

$$P = \frac{V_{\text{rms}}^2 R}{Z^2}$$

or $$200 = \frac{160^2 \times R}{80^2}$$

$$\therefore \quad R = 50\,\Omega$$

We know that, $Z = \sqrt{R^2 + X_L^2}$

or $$80 = \sqrt{50^2 + X_L{}^2}$$

$$\therefore \quad X_L = 62.5\,\Omega$$

The back emf, $V_L = iX_L = 2 \times 62.5$

$= 125\text{V}.$

6. **(100)** $X_L = \omega L = (2\pi \times 500) \times 8.1 = 25.4\,\Omega$

and $X_C = \frac{1}{\omega C} = \frac{1}{(2\pi \times 500) \times (12.5 \times 10^{-6})} = 25.4\ \Omega$

As $X_L = X_C$, so resonance will occur and $V_R = 100V$.

7. **(0.625)** Average power consumed is given by

$$P = \frac{V_{\text{rms}}^2 R}{Z^2},$$

where $$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + (2\pi \times \frac{30}{\pi} \times 0.4)^2}$$

$$= 26\,\Omega$$

$\therefore \quad P = \frac{6.5^2 \times 10}{26^2} = 0.625\,\text{W}$

8. **(0.8)** Power factor $= \cos\phi = \frac{R}{Z} = \frac{12}{15} = \frac{4}{5} = 0.8$

9. **(400)** $V = 220\text{V};\ r = 10\Omega$

$R' = 10 + 100\Omega = 110\Omega$

$I = \frac{V}{R'} = \frac{220}{110} = 2A$

$P = I^2R = 4 \times 100 = 400\ W$

10. **(5)** From Kirchoff's current law,

$i_3 = i_1 + i_2 = 3\sin\omega t + 4\sin(\omega t + 90°)$

$\Rightarrow i_3 = i_0 \sin(\omega t + \phi)$

where $i_0 = \sqrt{3^2 + 4^2 + 2(3)(4)\cos 90°}$

and $\tan\phi = \frac{4\sin 90°}{3 + 4\cos 90°} = \frac{4}{3}$

$\therefore\ i_3 = 5\sin(\omega t + 53°)$

11. **(25)** $f = \frac{1}{2\pi\sqrt{LC}} = \frac{2}{2\pi\sqrt{5 \times 80 \times 10^{-6}}} = \frac{25}{\pi}\ Hz$

12. **(2)** If first case, $\tan 60° = \frac{X_L}{R} = \frac{\omega L}{R}$

or $\quad \sqrt{3} = \frac{300L}{100}$

$\therefore \quad L = 0.58\ \text{H}$

In second case, $\tan 60° = \frac{X_C}{R} = \frac{1}{\omega CR}$

or $\quad \sqrt{3} = \frac{1}{300C \times 100}$

$\therefore \quad C = 19.2\,\mu\text{F}$

The impedance of the circuit is

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{100^2 + \left(300 \times 0.58 - \frac{1}{300 \times 19.2 \times 10^{-6}}\right)^2}$$

$$= 100\,\Omega$$

Current, $\quad i = \frac{V}{Z} = \frac{200}{100} = 2\text{A}$

13. **(0.08)** The resistance of the coil,

$$R = \frac{V}{i} = \frac{12}{4} = 3\Omega$$

Impedance, $Z = \frac{12}{2.4} = 5\Omega$

As $$Z = \sqrt{R^2 + X_L^2}$$

$\therefore$ $$X_L = 4\Omega$$

and $$L = \frac{X_L}{\omega} = \frac{4}{50} = 0.08\,\text{H}$$

14. **(133)** The capacitive reactance

$$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi \times 200) \times 12 \times 10^{-6}}$$

$$= 66.35\,\Omega$$

$$V_C = i_{rms} X_C = 2 \times 66.35$$
$$= 133\,\text{V}$$

15. **(0.704)** The resonance frequency,

$$\omega_r = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{10 \times 10^{-3} \times 1 \times 10^{-6}}}$$

$$= 10^4 \text{ rad/s}$$

Thus $$\omega = 0.90\,\omega_r = 9 \times 10^3 \text{ rad/s}$$

$$X_L = \omega L = 9 \times 10^3 \times (10 \times 10^{-3})$$

$$= 90\,\Omega$$

and $$X_C = \frac{1}{\omega C} = \frac{1}{9 \times 10^3 \times 10^{-6}}$$

$$= 111.11\,\Omega$$

The impedance, $Z = \sqrt{R^2 + (X_L \sim X_C)^2}$

$$= \sqrt{3^2 + (111.11 - 90)^2}$$

$$= 21.32\,\Omega$$

The current amplitude,

$$i_0 = \frac{V_0}{Z} = \frac{15}{21.32} = 0.704\text{ A}$$